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Discrete gauge symmetries, baryon number and large extra dimensions

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Abstract: Krauss and Wilczek have shown that an unbroken discrete gauge symmetry is respected by gravitationally mediated processes. This has led to a search for such a symmetry compatible with the standard model or MSSM that would protect protons from gravitationally mediated decay in a universe with a low scale for quantum gravity (large extra dimensions). The fact that the discrete symmetry must remain unbroken and have a gauge origin puts important restrictions on the space of possible discrete symmetries.

Keywords: Extra Large Dimensions, Discrete and Finite Symmetries, Black Holes.

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Contents

1. Introduction

Gravitationally mediated proton decay has long been postulated to result from the "spacetime foam" predicted in the current understanding of quantum gravity [1, 2, 3]. This process is completely negligible in traditional cosmologies, with the Planck scale safely relegated to extremely high energies ($M_{pl} \sim 10^{19}$ GeV). It has been proposed, however, that in a cosmology with the quantum gravity scale reduced by the presence of large extra dimensions [4], proton decay can proceed extremely rapidly [5].

One obvious way to circumvent this issue is to invent a new gauge symmetry that protects against effective operators that mediate fast proton decay. An unbroken gauge symmetry, however, would require a new massless gauge boson, which is phenomenologically unacceptable. A broken symmetry, on the other hand, is known to be violated by black hole physics in the classical regime [6]. For this reason, it is often assumed that the virtual Planck-scale black holes arising in a spacetime foam picture will similarly violate broken gauge symmetries [7, 8]. Thus, we are left with no way to protect protons against decay mediated by virtual black holes.

This difficulty was resolved by Krauss and Wilczek [9] who proposed that a surviving discrete remnant of a broken U(1) gauge symmetry could protect protons while at the same time allowing the associated gauge boson to gain a large mass. They showed that a gauge symmetry broken to a discrete remnant will still have an associated conserved surface integral which could be used to detect the presence of charge swallowed by a black hole.

Our purpose in this paper is to decide if there are any phenomenologically acceptable discrete gauge symmetries (arising from a U(1) gauge symmetry) that can protect against fast proton decay and remain unbroken in the low energy standard model or MSSM. We wish to proceed in a manner analogous to Ibáñez and Ross's work [11], in which they constructed a catalog of discrete symmetries that protect against SUSY-mediated proton decay.

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2. The setting

Classical black holes fail to respect a broken gauge symmetry regardless of the energy scale that characterizes the breaking [6]. By analogy, therefore, it has been assumed that quantum black holes also fail to respect any broken symmetry [3, 7, 8]. By exploiting this argument, Hawking, Page and Pope showed that a spacetime foam of virtual black holes can lead to an effective four-point interaction converting two quarks into a lepton plus anti-quark [3]. This interaction will be suppressed by inverse powers of the Planck mass, so that for $M_{pl} = 10^{19}$ GeV, the predicted rate of proton decay is well below experimental bounds. These authors also examined the effect of spacetime foam on the masses of particles. Their results indicate that quantum gravity effects will not produce fermion and vector boson masses. Scalars, on the other hand, were shown to acquire a mass of order the Planck mass, which poses a problem for the Higgs mechanism of electroweak symmetry breaking.

More recently, with the advent of theories with a low scale for quantum gravity [4], the spacetime foam picture been revisited. In models with one or more large extra dimensions, it is a desired feature that the quantum gravity scale be nearly equal to the scale for electroweak symmetry breaking so as to solve the hierarchy problem. This device eliminates the problem of large scalar masses in the spacetime foam picture as well, since the contributions from gravitational effects are reduced along with the Planck scale. This scenario is not completely beneficial, however, since lowering the Planck scale only serves to make the four-point operators which produce proton decay more important (less suppressed). Calculations show that the proton lifetime can be a very important constraint on theories with large extra dimensions if we assume the spacetime foam picture is correct. In fact, the current experimental bounds on proton decay would require the scale for quantum gravity to be high enough so as to be unreachable in any current or proposed collider [5].

For this reason, it is of interest to find a discrete symmetry that forbids proton decay (or at least the dimension six operators that result in fast proton decay), can remain unbroken at low energies, and could potentially arise as the remnant of a gauged symmetry. All three conditions are necessary to protect protons in the spacetime foam picture with a low scale for quantum gravity. This leads to the constraint that the electroweak Higgs(es) must be uncharged with respect to any discrete gauge symmetry that is to protect protons against decay. Otherwise, the symmetry is broken today and its protection is lost.

If we want to retain the Higgs mechanism for fermion masses, an uncharged Higgs leads directly to a constraint on the discrete charges of the known fermions. We would like to implement these constraints on the Ibáñez-Ross (IR) catalog of discrete symmetries [11]. Unfortunately, their work assumes that hypercharge is unbroken. This is unacceptable if we are to find a symmetry that remains unbroken after the electroweak Higgs gets a VEV. We will, however, borrow from their terminology. In the IR language, we can easily construct the constraints that apply to a discrete symmetry that is unbroken today. The first constraint is provided by fermion mass terms:

$$\alpha_{Q_i} + \alpha_{u_i} = 0 \bmod N \tag{2.1}$$

$$\alpha_{Q_i} + \alpha_{d_i} = 0 \bmod N \tag{2.2}$$

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$$\alpha_{L_i} + \alpha_{e_i} = 0 \bmod N \tag{2.3}$$

$$\alpha_{L_i} + \alpha_{\nu_i} = 0 \bmod N \tag{2.4}$$

where each α represents the integer charge of the (super)field noted under the discrete gauge group of interest and i is a family index (ranging from one to three).

Before moving on, we should comment on neutrino masses. Note that relation (2.4) would be necessitated by Dirac masses. Many of the popular scenarios for neutrino mass in a universe with large extra dimensions utilize Dirac neutrinos [14]. It is, however, possible that neutrinos possess only Majorana mass terms. In effect, this case is actually more restrictive. Equation (2.3) is unmodified. Further, we would be forced to constrain α_L to equal $N/2$ to allow Majorana terms without further breaking our $\mathbb{Z}_N$ symmetry. As we shall see, the conditions above are already sufficient to place significant constraints on the space of allowed $\mathbb{Z}_N$ symmetries, and so we will not assume neutrino Majorana masses are allowed.

We can express the restrictions of equations (2.1)–(2.4) very succinctly (again, following the terminology of [11]):

$$\vec{\alpha}_i = (\alpha_{Q_i}, -\alpha_{Q_i}, -\alpha_{Q_i}, \alpha_{L_i}, -\alpha_{L_i}, -\alpha_{L_i}) \tag{2.5}$$

where $\vec{\alpha}_i$ is a shorthand way of denoting the charges of each family of (super)fields under the action of a discrete symmetry. By definition:

$$\vec{\alpha}_i \equiv (\alpha_{Q_i}, \alpha_{u_i}, \alpha_{d_i}, \alpha_{L_i}, \alpha_{e_i}, \alpha_{\nu_i}) \tag{2.6}$$

(note that we have broken from [11] by eliminating α_H (since it has been set to zero), and by adding α_ν and a family index i).

This is a dramatic restriction, as it means the discrete symmetries which can remain unbroken today are parameterized by six integers (assuming three families): α_{Q_i} and α_{L_i}. In fact, we can be even more restrictive by using the experimental observations that quarks and neutrinos mix among the families. This implies that we can do away with the family index. Any symmetry that remains unbroken today must be family independent. (Family-dependent symmetries are often used in model building to give approximately correct CKM mixing or neutrino mixing parameters. Such models, however, rely on the presence of one or more Higgs-type fields that are charged under the symmetry. We can have no such Higgs without breaking our symmetry and losing its protection.) Thus, we have limited the possible discrete symmetries to those under which all families have the charges:

$$\vec{\alpha} = (\alpha_Q, -\alpha_Q, -\alpha_Q, \alpha_L, -\alpha_L, -\alpha_L)\,. \tag{2.7}$$

3. Forbidding fast baryon number violation

To forbid fast proton decay, our discrete symmetry must forbid the operators $uude$, $QQQL$ and (perhaps) $udd\nu$. Each of these results in the same constraint on the charges α_Q and α_L:

$$3\alpha_Q + \alpha_L \neq 0 \bmod N\,. \tag{3.1}$$

The above condition is all that is required to forbid fast gravitationally mediated proton decay. There is, however, one additional signature of baryon violation which has been experimentally investigated. That process is neutron-antineutron oscillation, which changes the baryon number of the universe by $\Delta B = 2$. Such oscillations would be mediated by effective six-quark operators of the form *uddudd*.

The experimentally observed lack of $\Delta B = 2$ processes can also serve as an important constraint on models with low Planck scale [5]. Current limits would require that the scale for quantum gravity be restricted to $M_{qg} > 10^5\text{GeV}$ [15]. Thus, if we hope to observe the effects of quantum gravity at the LHC or indeed to solve the hierarchy problem with minimal fine-tuning, we must forbid neutron-antineutron oscillation as well as proton decay. To accomplish this, we must impose the inequality:

$$6\alpha_Q \neq 0 \bmod N\,. \tag{3.2}$$

Equations (3.1) and (3.2) are important constraints. Equation (3.2) alone eliminates any unbroken $\mathbb{Z}_N$ symmetry with $N = 2, 3$ or 6 from consideration as a means to prevent fast baryon number violation in a universe with a low scale for quantum gravity. It also eliminates the possibility that $\alpha_Q = 0$. Adding the constraint of equation (3.1) leaves only a handful of possibilities with $N \leq 6$. These are listed in table 1.

N	α_Q	α_L
4	1	0
4	1	2
4	1	-1
5	1	0
5	1	1
5	1	-2
5	1	-1
5	2	0
5	2	1
5	2	2
5	2	-2

Table 1: Charges α_Q and α_L under all possible (independent) unbroken $\mathbb{Z}_N$ symmetries with $N \leq 6$ which satisfy equations (3.1) and (3.2).

4. Anomalies

Further constraints on the charges α_Q and α_L could be obtained by considering discrete gauge anomalies [10, 13]. Unless the anomalies cancel, it is impossible for the discrete symmetry to be a remnant of a gauged U(1).

Unfortunately, discrete anomalies are notoriously ambiguous [12, 13]. Theoretically, we should have several anomalies to consider. The U(1) giving rise to our discrete symmetry (henceforth $\mathrm{U_D}(1)$) must be compatible with the SU(3), SU(2) and $\mathrm{U_Y}(1)$ of the standard model, giving rise to possible (nontrivial) anomalies of the form: $\mathbb{Z}_N^3$, $\mathbb{Z}_N^2 \times \mathrm{U_Y}(1)$, $\mathbb{Z}_N \times \mathrm{U_Y}(1) \times \mathrm{U_Y}(1)$, $\mathbb{Z}_N \times \mathrm{SU}(2) \times \mathrm{SU}(2)$, $\mathbb{Z}_N \times \mathrm{SU}(3) \times \mathrm{SU}(3)$, and the mixed gravitational anomaly. We will consider each in turn.

The $\mathbb{Z}_N^3$ is basically non-constraining due to the possible existence of heavy fermions that are fractionally charged under the remnant $\mathbb{Z}_N$ discrete symmetry [12].

The mixed hypercharge anomalies are similarly difficult to evaluate due to the unknown relative normalizations of the $\mathrm{U_D}(1)$ and $\mathrm{U_Y}(1)$ groups [10].

The mixed SU(3) anomaly is trivial. Without exotic fermions it is proportional to:

$$2 \times 3\alpha_Q + 3\alpha_u + 3\alpha_d = 6\alpha_Q - 6\alpha_Q = 0 \tag{4.1}$$

so that the anomaly cancels for any choice of α_Q and α_L.

The gravitational anomaly is similarly trivial.

The most promising anomaly constraint is provided by the mixed SU(2) anomaly. Here, extremely massive fermions cannot contribute because the symmetry protects the masses. We will therefore assume that the only fermions charged under the weak SU(2) symmetry of the standard model are are the three known families of quarks and leptons. In that case, the discrete $\mathbb{Z}_N \times \mathrm{SU}(2) \times \mathrm{SU}(2)$ will be proportional (mod N) to:

$$3 \times 3\alpha_Q + 3\alpha_L = 9\alpha_Q + 3\alpha_L\,. \tag{4.2}$$

Direct cancellation of the SU(2) anomaly would then require:

$$9\alpha_Q + 3\alpha_L = 0 \bmod N. \tag{4.3}$$

There is, however, one other possibility to consider.

Anomaly cancellation can also be achieved via a version of the Green-Schwarz (GS) mechanism [16]. In heterotic string theory, the GS mechanism is basically model independent, since it involves only the dilaton superfield. In type-II string theories with D-branes and orientifolds (or type-I string theories) which can accommodate a low string scale, on the other hand, the GS mechanism is generalized because of the presence of Ramond-Ramond (RR) 2-forms [17]. Thus, if we construct such a theory which involves M RR 2-forms B^i ($i = 1, 2, \ldots, M$) we may expect terms in the lagrangian of the form [18]:

$$\mathcal{L}_{GS} = \sum_{i=1}^{M} c_i B^i \wedge F_D + d_i \eta^i \,\mathrm{tr}\left(F_{\mathrm{SU}(2)} \wedge F_{\mathrm{SU}(2)}\right) \tag{4.4}$$

where η^i is the scalar dual to B^i, F_D is the field strength associated with the $\mathrm{U}_D(1)$ gauge boson, and $F_{\mathrm{SU}(2)}$ is the field strength associated with the SU(2) group (we mention only the SU(2) group because it alone yields a nontrivial mixed $\mathrm{U}_D(1)$ anomaly — the couplings can be present for any group). The model-dependent coefficients c_i and d_i will then make a contribution to the mixed $\mathrm{U}_D(1)\times\mathrm{SU}(2)\times\mathrm{SU}(2)$ anomaly [19]:

$$\delta A = \sum_{i=1}^{M} c_i d_i\,. \tag{4.5}$$

Currently, there are no generic constraints on the values of the coefficients c_i and d_i. Thus, if the GS mechanism is applicable, it removes the constraint of equation (4.3).

It is important to remember, however, that the GS mechanism apparently breaks the associated anomalous U(1) symmetry down to a global symmetry [16, 20]. This has been used as a means to protect protons in string theory [21], but in the case of virtual black hole mediated decay a residual global U(1) is not sufficient protection. Instead, the gauge symmetry must be broken in a way that still admits nontrivial strings or vortex solutions [9, 22]. The GS mechanism breaks the associated U(1) via the Stückelberg mechanism [23], which does not obviously admit such strings. Of course, this is hardly conclusive. It has recently been asserted that the GS mechanism does not prohibit string configurations, and postulated that all discrete symmetries in string constructions are gauged discrete

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p	Allowed N
1	3
2	3, 6
3	9
4	6, 12
5	15
6	9, 18
7	21
8	12, 24
9	27
10	15, 30

Table 2: Values of N which satisfy the constraint of equation (4.3) for the given values of $p \equiv 3\alpha_Q + \alpha_L \bmod N$.

N	α_Q	α_L	p
9	1	0	3
9	1	3	6
9	2	0	6
9	2	6	3
9	4	0	3
9	4	3	6

Table 3: Independent combinations of α_Q and α_L allowed by constraints (3.1), (3.2) and (4.3) for $N = 9$ (the only allowed $N \leq 10$).

symmetries [24]. For the purposes of this work, we will consider this an open question. We therefore catalog the constraints that would become available if it is shown that the GS mechanism *does not* give rise to gauged discrete remnants. If the GS mechanism is shown to allow discrete gauge symmetries, we are left with the (still quite restrictive) constraints given in section 3.

In the case that the GS mechanism is not available, we must impose the constraints of equations (3.1), (3.2) and (4.3) to decide what values of α_Q and α_L are allowed for a given $\mathbb{Z}_N$ symmetry. For convenience, we now define the quantity $p \equiv 3\alpha_Q + \alpha_L$. With this definition, equation (3.1) requires $p \neq 0 \bmod N$ and equation (4.3) becomes $3p = 0 \bmod N$. These two equations for p allow us to find the allowed N values corresponding to any given p. Table 2 shows the possible N values for p's up to 10. We next add the constraint of equation (3.2). In table 3, we report the only (independent) α_Q and α_L charges which satisfy all three constraint equations for $N \leq 10$. This table reports the only possible (independent) $\mathbb{Z}_N$ discrete symmetries with $N \leq 10$ which will remain unbroken and forbid gravitationally mediated proton decay and gravitationally mediated neutron-antineutron oscillation in the case that the GS mechanism is *unavailable* for anomaly cancellation. Note that under our assumptions — three families of quarks/leptons and Dirac mass terms for neutrinos (recall that Majorana masses tend to be *more* restrictive) – any $\mathbb{Z}_N$ arising from a U(1) gauge symmetry which is to remain unbroken and protect against fast gravitationally-mediated baryon violation in a universe with a low scale for quantum gravity must have $N \geq 9$ if the GS mechanism is unavailable.

5. Conclusions

If we assume by analogy with classical physics that broken symmetries are incapable of protecting against gravitationally mediated proton decay and neutron-antineutron oscillation, we are led to the constraint that the electroweak Higgs(es) must be neutral under any

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surviving discrete gauge symmetry that would be used to protect baryon number. This simple constraint, when coupled with the assumption of three families of quarks and leptons and Dirac neutrino masses, is sufficient to eliminate all $\mathbb{Z}_N$ symmetries arising from a gauged U(1) with $N = 2, 3$ or 6 from consideration. Perhaps more importantly, unbroken $\mathbb{Z}_N$ symmetries are parameterized by only two independent charges.

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